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Comparison between continuous sensitivity analysis and Taguchi method in optimization of electromechanical devices

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Abstract
In this paper, the optimization of electromechanical devices by continuous sensitivity analysis and its comparison with Taguchi method is investigated. The present research can be useful for researchers in one of the important and practical issues that is the optimization of different systems. In practice, it will be observed that the continuous sensitivity analysis method, in addition to achieving significant results, also increases the convergence rate significantly compared with the Taguchi method. In addition to the above, this dissertation also tries to improve the method of continuous sensitivity analysis, which has been done using the help of interpolation such as Spline and Bizar. One of the advantages of the continuous sensitivity analysis method using Spline interpolation in comparison with the Taguchi method is the absence of sharp angles at the optimized border and ease of construction. This specification is of great importance in electromagnetic devices. The main purpose of this article is to compile a complete and understandable reference for use in today's industry and further studies.

Keywords: Electromechanical optimization, Machine design, Taguchi method, Electromagnetism, Sensitivity.

1- Introduction

As a tool for analyzing the electromagnetic systems, the finite element method (FEM) is a well-known method for determination of the field distribution to evaluate their performance.

Using the geometry of a system and the properties of materials, the field distribution can be calculated numerically using FEM. Many times, however, such an analysis is not enough and it is necessary to find the best design for an issue according to the existing needs. Issues such as these are called optimal design problems. Since objective functions in shape optimization are typically nonlinear and implicit functions of design variables, sensitivity analysis and an adjunct variable method can be used. Sensitivity is defined as the relative change in performance to the design variable and is expressed by the derivative of the whole objective function relative to the design variable.

Accurate calculation of sensitivity is a fundamental and very important step to formulate a mathematical model for the optimization problem and provides the designer with the relevant gradient information for the search direction. Using this gradient, the shape optimization method can be done with the help of programming. There are two methods for calculating the sensitivity, which can be numerical differentiation and analytical differentiation [1]. Analytical differentiation itself includes two methods, discrete and continuous. In the discrete method, the sensitivity is obtained by differentializing the design variable of the system algebraic equation [1]. The algebraic equation of the system is also obtained by finite element method by separating the analysis area. Because in this method, the position of the nodes is considered as a design variable, it is applicable to many systems that can be analyzed by the discrete method. But the disadvantage is that in some modeling, such as fundamental functions, it leads to complex formulations and makes it necessary to access finite element codes to calculate the sensitivity.

In the continuous method, we obtain the exact sensitivity formula from the Werdash equation that is not discretized [1], [2] and [3]. In [4], the effect of the type and setting parameters and determination of optimal levels in electrical discharge machining of alloy DIN 1.2080 using the Taguchi method is examined. In [5], the influence of different parameters on equal channel angular pressing (EADAP) of titanium alloy is investigated using Taguchi method. Authors in [6] used Taguchi method In order to explore the influences of shielding layer material, shielding layer thickness and via spacing on System in Package (SiP) on Electric field Shielding Effectiveness (SE) in conformal shielding structure, and to reduce simulation time cost. The sensitivity formula is expressed as a dual integral along the common boundary of the shape, and it is an exact function of the state and adjunct variables at the common boundary. Since it is usually impossible to obtain exact state and adjunct variables using existing finite element codes, we can obtain approximate state and adjunct variables and use them in the formula to numerically evaluate the sensitivity. In this method, since the derivative of the state variable to the design variable is obtained from the non-discrete equation, there is no need for any method to calculate the derivative of the state variable to the design variable, unlike the discrete approach, and all we have to do is to program for Numerical calculation of line integral is a sensitivity formula.

2- Sensitivity Formula

In an electromagnetic system, the homogeneous boundaries of Dirichlet and Newman (outer boundaries) are usually due to the symmetry of the system geometry and the distribution of the source. Generally, these borders do not have a direct role in the optimal design of the shape. Shape design parameters are found in internal boundaries, both sides of which have different physical characteristics such as current density, permeability and permanent magnetization. Therefore, the sensitivity formula can be applied to almost all shape design problems in magnetism due to changes in internal boundaries. The formula of sensitivity is obtained by using the concept of continuous derivative and an adjunct variable method, the proof of which is beyond the scope of this paper and can be fully studied in [1]. From (1) and (2) we can calculate the derivative of the objective function relative to the design parameter.

$$\frac{dF}{dP} = \int_{\Gamma}^{0} G(A,\lambda) n^T \frac{\partial x(p)}{p} d\Gamma \quad (1)$$

$$G(A,\lambda) = B(\lambda^{**})^T \left\{ (V^* - V^{**}) B(A^*) \right\} \quad (2)$$

In Eqs. (1) and (2), F represents the objective function, p is the design parameter and Γ stands for the moving boundary, while n is the normal vector to the outside of region 1, V represents the magnetic resistance, m stands for the permanent magnetization, j is the current density, and * and ** represent the regions 1 and 2 respectively. These regions surround the common border.

In Fig. 1, the properties of the materials * and ** are the same. The common boundary cannot be modified and there is no need for integration in that part. Only at the integral boundary is the sensitivity valid that the two sides of the common boundary have different properties. It should be noted that in the sensitivity formula, changing the two regions * and ** has no effect on the formula in the sense that it reverses the direction of the vector n

and changes the physical properties, so its sign is not changed. It should be noted that the interchangeability of * and ** means that the direction of the vector n can be optionally defined in any integral component until the region * is defined in the direction of the vector n. This makes it easier to numerically integrate the sensitivity formula.

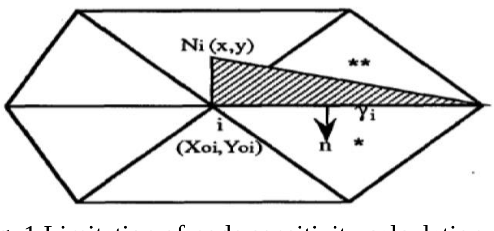

Fig. 1 Limitation of node sensitivity calculation

3- Procedure of the Sensitivity Calculation

First, the analysis model for the optimal design is initially defined by the finite element model, loads, design parameters, and the objective function. The finite element method is then used to calculate the state variable; After obtaining the state variable, an additional load for the objective function is calculated externally for the finite element and given to the finite element; Finally, using the obtained state and additive variables, the sensitivity is obtained by numerical integration of the sensitivity formula. In the above process, it is easy to calculate the additional load and state according to the available finite element codes and they can be obtained. [1-3], [9] and [10].

4- Numerical Calculation of Sensitivity

As shown in Fig. 1, the moving boundary consists of a series of node points (x_{oj} and y_{oj}) and the sensitivity is related to the node points on the boundary and can be expressed by the sensitivity of the design parameters as follows.

$$\frac{df}{dp} = \sum_{i=1}^{n} \left(\frac{\partial X}{\partial X_{oi}} \frac{\partial X_{oi}}{\partial p} + \frac{\partial F}{\partial Y_{oi}} \frac{\partial Y_{oi}}{\partial p} \right) \quad (3)$$

Where n is the total number of nodes on our correction boundary.

$$\frac{\partial F}{\partial X_{oi}} = \int_{\Gamma}^{0} G(A,\lambda) n^T \frac{\partial X}{\partial X_{oi}} d\Gamma \quad (4)$$

$$\frac{\partial F}{\partial Y_{oi}} = \int_{\Gamma}^{0} G(A,\lambda) n^T \frac{\partial X}{\partial Y_{oi}} d\Gamma \quad (5)$$

The modifiable boundary can be interpolated as formula (6) using a weighting function $N_i(x,y)$

$$X = (x, y)^T = \sum_{j=1}^{n} (X_{oj}, Y_{oj})^T N_j(x, y) \quad (6)$$

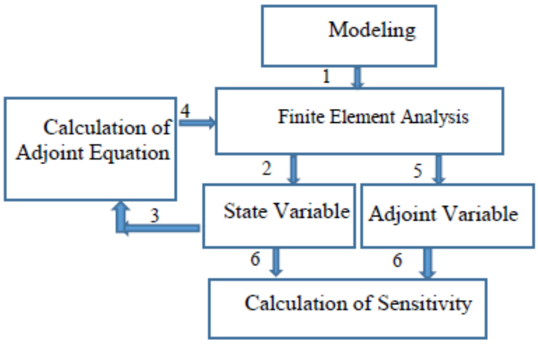

Fig. 2 Calculation of Sensitivity Analysis

It should be noted that there is no need for the exact same weighting function with the function in finite element modeling because the sensitivity calculation is performed externally to finite element codes [2].

$$n^T \frac{\partial X}{\partial X_{oj}} = n_x N_j(x, y) \quad (7)$$

N_j and n_x are components of y, x are vectors of n, respectively. Therefore, the sensitivity is rewritten based on (4), (5), (8) and (9).

$$\frac{\partial F}{\partial X_{oi}} = \int_{\Gamma}^{0} G(A, \lambda) n_x N_j d\Gamma \quad (8)$$

$$\frac{\partial F}{\partial Y_{oi}} = \int_{\Gamma}^{0} G(A, \lambda) n_y N_j d\Gamma \quad (9)$$

Now we can say that the sensitivity of node i is obtained by numerical integration of the sensitivity formula of (10).

5- Optimization algorithm

By analyzing the mentioned sensitivity, any iterative optimization algorithm can be used. In this paper, a gradient image method is used to numerically solve the optimal design problem, to find a design that minimizes the objective function despite the limitations.

This method is summarized as follows: first, the direction of the steepest descent is determined by evaluating the sensitivity formula. In this case, a small movement in the resulting direction will reduce the objective function. When the objective function is nonlinear with respect to the design parameter, the motion can cause a very small deviation from the constraint, but by using the gradient information this constraint is compensated. The above process is repeated until the objective function converges.

6- Relationship between design parameters and node points

In the design of electromagnetic equipment, if the design parameters correspond one by one to the node points of the finite element, the shape of the saw tooth is obtained and accuracy problems can be created in the design. In order to solve this problem, designs in the parameter Is bounded by the relation of the design variable and the node points of the element. In (3) derivatives of node points according to the design parameter are expressed as (10).

$$\left(\frac{\partial X_{oi}}{\partial p}, \frac{\partial Y_{oi}}{\partial p}\right) = (\alpha_i, \beta_i) \quad (10)$$

For example, when a moving boundary is composed of several node points and is linearly interpolated by the two design parameters n and m as shown in Fig. 3, [2].

The objective function in this optimization is the energy of the electromagnetic system, which can be written as follows:

$$F = W_o - W_i \quad (11)$$

W_i is the total energy in the magnetic system in the initial state and W_o is the total energy in our optimized system. Our goal is to reduce the function to its lowest value. According to the objective function, our design variable for this work is the flux density in the relevant device [3] [1] and since the objective function is based on energy, there will be no need to solve and obtain the additional variable [3]] And the design variable is equal to the add variable. The moving boundary or the same correction boundary is shown in Fig. 6, in the correction boundary three design parameters with coordinates (3.5 and 0.15), (3.5 and 0.3) and (3.5 and 45) / 0) There. The system is transformed into 4834 triangular meshes. At all stages, the current density in the coil is set to 200,000. After comparing and obtaining the results, the energy value of the system W_i = 197460J is obtained. Now, according to what has been said, the optimization steps are done step by step, and we use the spline method to parameterize it.

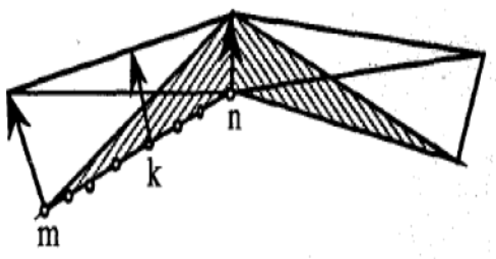

Fig. 3 Definition of the node parameters

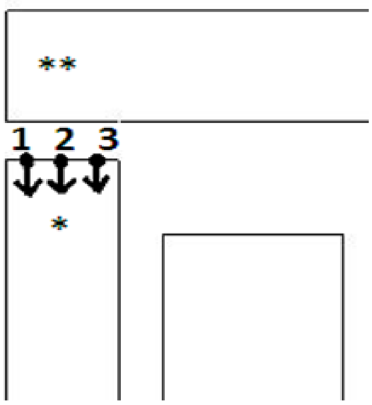

Fig. 4 Three design parameters

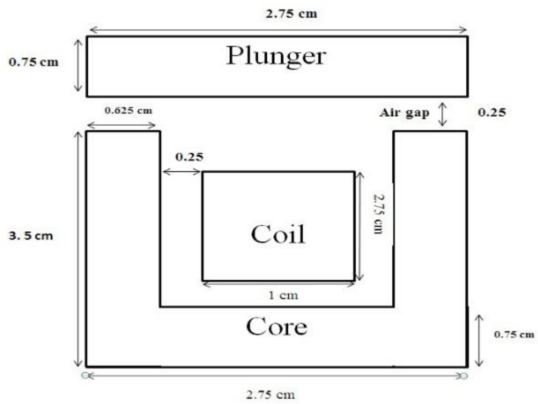

Fig. 5 The first modeling

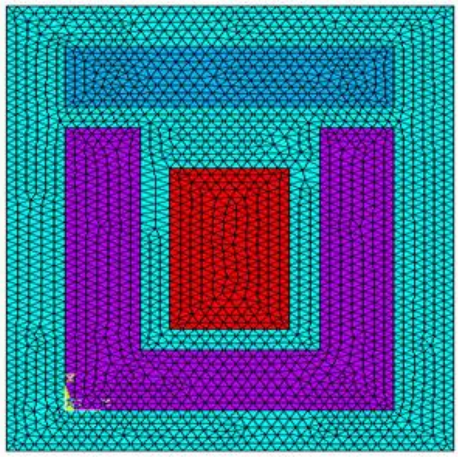

Fig. 6 Meshing of the model

Table 1: Design variables coordinates

Level	Point 1		Point 2		Point 3	
	X	Y	X	Y	X	Y
1	0.15	3.476	0.3	3.47	0.45	3.48
2	0.149	3.458	0.3	3.46	0.45	3.47
3	0.147	3.45	0.3	3.44	0.146	3.46

The optimization steps are performed in 7 steps, in which only 3 significant changes were seen and in the later stages, no significant change was seen by being in a repeating cycle. The final optimized shapes and results related to the objective function can be seen below.

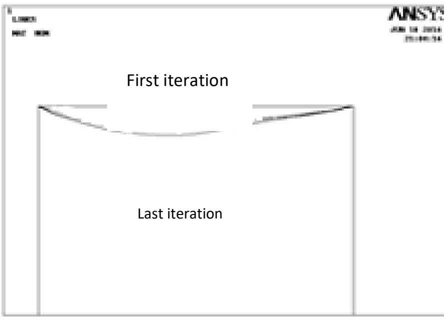

Fig. 7 Boundary shaping of the model

Table 2: Results of the objective function

Level	Wi	Wo	Wo-Wi
1	197460	197780	320
2	197780	197900	120
3	197900	197930	30

7- Comparison of continuous sensitivity analysis with Taguchi method

One of the methods used to overcome the problem of large number of iterations in the optimization process is the Taguchi method. This method was developed by a Japanese engineer named Taguchi for efficient and practical optimization. In this method, instead of examining all possible modes for combining parameters, only specific modes are considered. In Taguchi optimization, like other optimization methods, the objective function is first defined according to the optimization goal. Then the design parameters affecting the defined objective function are determined. Next, for each parameter, several possible values are considered called levels.

The greater the number of levels for each parameter, the greater the accuracy of the operation and, in turn, the longer it takes for the optimization process. Therefore, time and hardware constraints are an important factor in determining the levels for each parameter. After selecting the parameters and their levels, the appropriate Taguchi table must be selected. Taguchi tables are easily accessible through references, and according to the number of parameters and the number of levels selected for them, one of the Taguchi tables is selected.

Considering that the Taguchi method is currently one of the best and most widely used methods in the industry, a simple comparison can show the advantages of the continuous sensitivity analysis method in this method.

If we pay attention to the simulation result in continuous sensitivity analysis, it becomes clear that our desired energy has been significantly optimized. The optimization was done in three stages and its speed was high. It should be noted that no initial guess is required at any stage, so unlike many methods, the designer does not need previous experience and information. Now, if the same example is optimized by the Taguchi method, it will be seen that according to the existing tables of orthogonal arrays, 18 experiments are needed, so it can be said that up to this point, the speed of sensitivity analysis is much higher and It should be said that in Taguchi method, if the initial guess is wrong, the designer will go too far from the optimization path. Another disadvantage of Taguchi over sensitivity analysis is that it requires previous designer experience.

8- Conclusion

According to the presented results, the present article shows the feasibility and numerical range of implementing sensitivity analysis with existing finite element codes. Critics of this method, because the border is designed by this method, it is a saw tooth

Was considered an impractical method, but this problem can also be solved by using

the spline interpolation method.
One of the advantages of this method is that the variables are independent of each other and the convergence speed is increased. Therefore, sensitivity analysis in the design of electromagnetic devices is expected to provide a major breakthrough that cannot be achieved by classical design methods.

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